Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7405

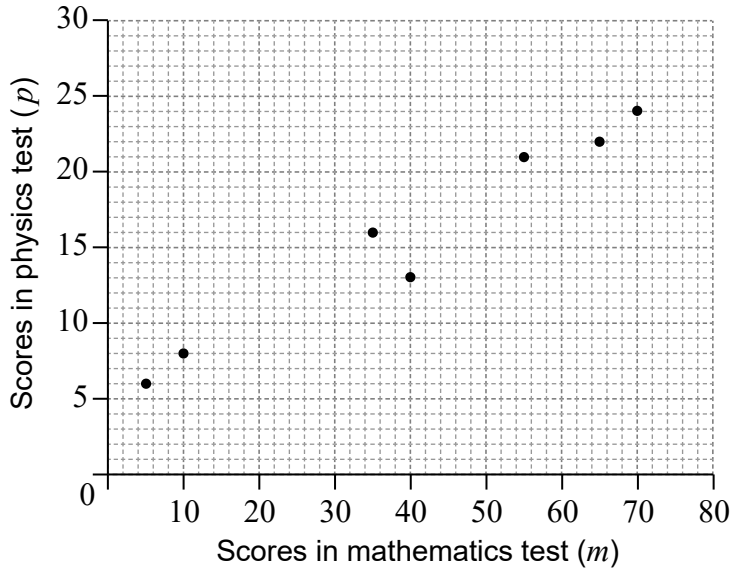
21 pages

Please **do not** write on this page.

Answers written on this page
will not be marked.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The following scatter diagram shows the scores obtained by seven students in their mathematics test, m, and their physics test, p.



The mean point, M, for these data is $(40, 16)$.

(a) Plot and label the point $\mathrm{M}(\bar{m}, \bar{p})$ on the scatter diagram. [2]

(b) Draw the line of best fit, by eye, on the scatter diagram. [2]

(c) Using your line of best fit, estimate the physics test score for a student with a score of 20 in their mathematics test. [2]

Working:

Answer:

(c) . . . . . .

2. Consider the following propositions.

p : the baby cries
q : the baby is happy
r : the baby wants to play

(a) Write down, in words, $(q \wedge r) \Rightarrow \neg p$. [3]

(b) Complete the following truth table. [2]

p	q	r	$\neg p$	$(q \wedge r)$	$(q \wedge r) \Rightarrow \neg p$
T	T	T	F		
T	T	F	F		
T	F	T	F		
T	F	F	F		
F	T	T	T		
F	T	F	T		
F	F	T	T		
F	F	F	T		

(c) State whether $(q \wedge r) \Rightarrow \neg p$ is a tautology, contradiction or neither. [1]

Working:

Answers:

(a) . . . . . .

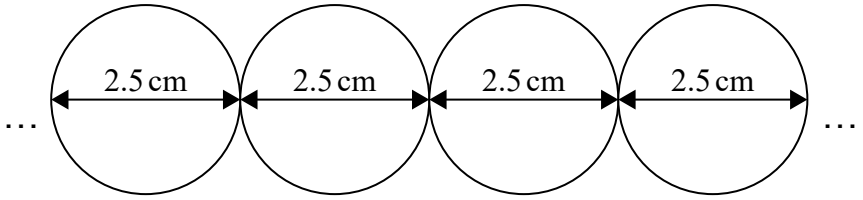
. . . . . .

. . . . . .

(c) . . . . . .

3. Last year a South American candy factory sold 4.8×10^8 spherical sweets. Each sweet has a diameter of $2.5\,\text{cm}$.

The factory is producing an advertisement showing all of these sweets placed in a straight line.



(a) Find the length, in cm, of this line. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [3]

The advertisement claims that the length of this line is x times the length of the Amazon River. The length of the Amazon River is $6400\,\text{km}$.

(b) (i) Write down the length of the Amazon River in cm.

(ii) Find the value of x. [3]

Working:

Answers:

(a) . . . . . .

(b) (i) . . . . . .

(ii) . . . . . .

4. The following table shows four different sets of numbers: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

Set	Example of a number in the set
$\mathbb{N}$	
$\mathbb{Z}$	
$\mathbb{Q}$	
$\mathbb{R}$	

(a) Complete the second column of the table by giving **one** example of a number from each set. [4]

Josh states: “Every integer is a natural number”.

(b) Write down whether Josh’s statement is correct. Justify your answer. [2]

Working:

Answer:

(b) . . . . . .

. . . . . .

. . . . . .

5. In this question, give all answers to two decimal places.

Karl invests 1000 US dollars (USD) in an account that pays a nominal annual interest of 3.5%, **compounded quarterly**. He leaves the money in the account for 5 years.

(a) (i) Calculate the amount of money he has in the account after 5 years;

(ii) Write down the amount of **interest** he earned after 5 years. [4]

Karl decides to donate this **interest** to a charity in France. The charity receives 170 euros (EUR). The exchange rate is $1\,\text{USD} = t\,\text{EUR}$.

(b) Calculate the value of t. [2]

Working:

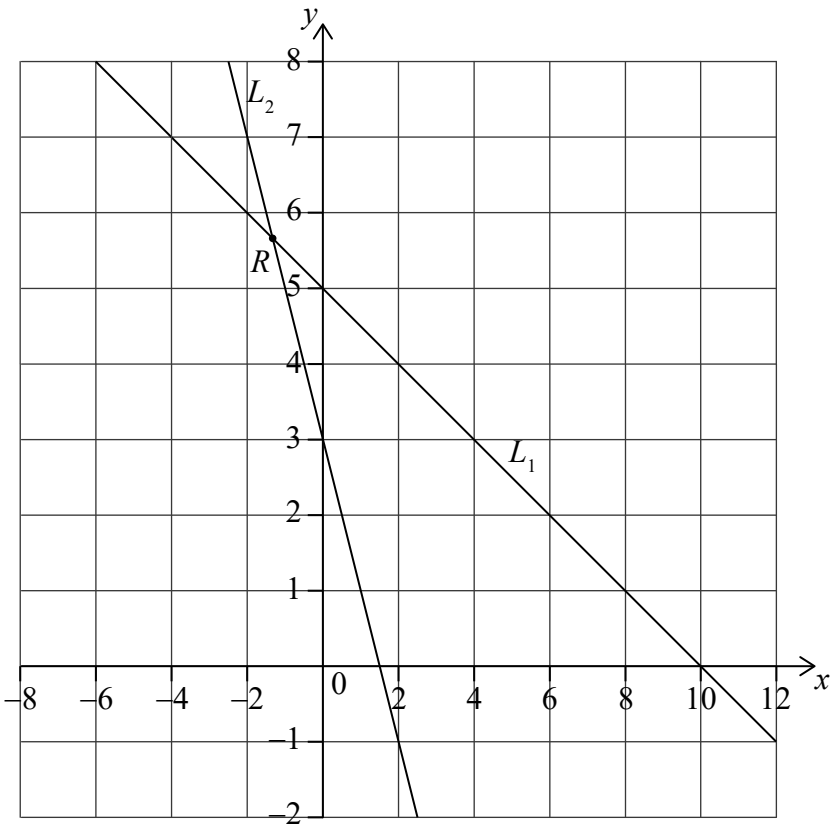
Answers:

(a) (i) . . . . . .

(ii) . . . . . .

(b) . . . . . .

6. Consider the straight lines L_1 and L_2. R is the point of intersection of these lines.



The equation of line L_1 is $y = ax + 5$.

(a) Find the value of a. [2]

The equation of line L_2 is $y = -2x + 3$.

(b) Find the coordinates of R. [2]

Line L_3 is parallel to line L_2 and passes through the point $(2, 3)$.

(c) Find the equation of line L_3. Give your answer in the form $y = mx + c$. [2]

(This question continues on the following page)

(Question 6 continued)

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

7. In an international competition, participants can answer questions in **only one** of the three following languages: Portuguese, Mandarin or Hindi. 80 participants took part in the competition. The number of participants answering in Portuguese, Mandarin or Hindi is shown in the table.

		Languages			
		Portuguese	Mandarin	Hindi	**Total**
Participants	Boys	20	18	5	**43**
	Girls	18	7	12	**37**
	Total	**38**	**25**	**17**	**80**

(a) State the number of boys who answered questions in Portuguese. [1]

A boy is chosen at random.

(b) Find the probability that the boy answered questions in Hindi. [2]

Two girls are selected at random.

(c) Calculate the probability that one girl answered questions in Mandarin and the other answered questions in Hindi. [3]

Working:

Answers:

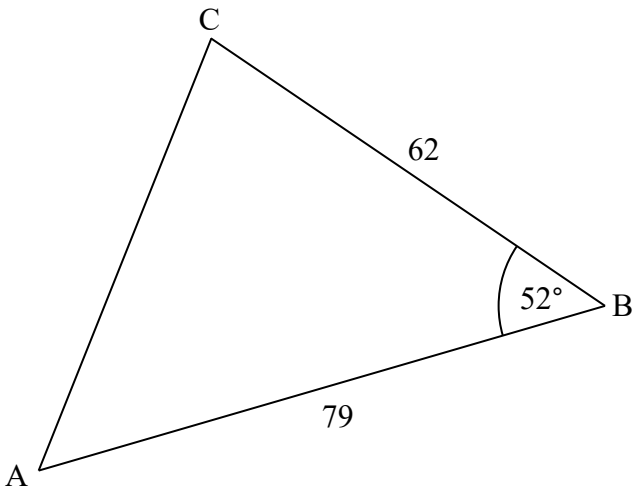
(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

8. A park in the form of a triangle, ABC, is shown in the following diagram. AB is $79\,\text{km}$ and BC is $62\,\text{km}$. Angle $A\hat{B}C$ is 52°.

diagram not to scale



(a) Calculate the length of side AC in km. [3]

(b) Calculate the area of the park. [3]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

9. Consider the following Venn diagrams.

Diagram 1

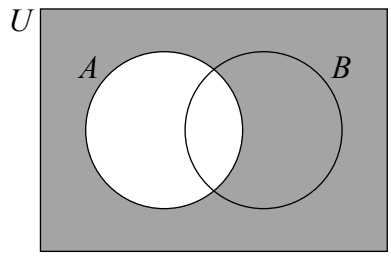


Diagram 2

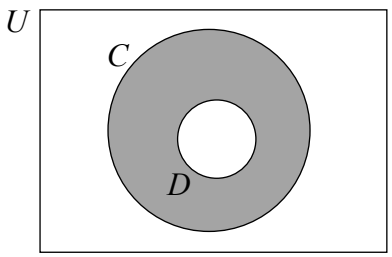
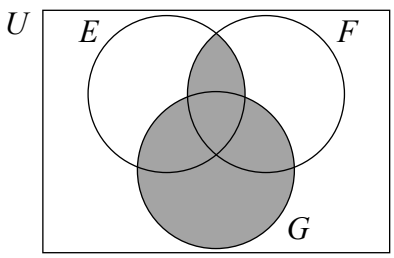


Diagram 3



(a) Write down an expression, in set notation, for the **shaded** region represented by

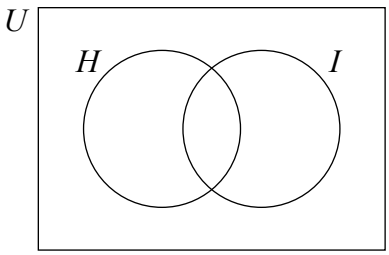
(i) Diagram 1;

(ii) Diagram 2;

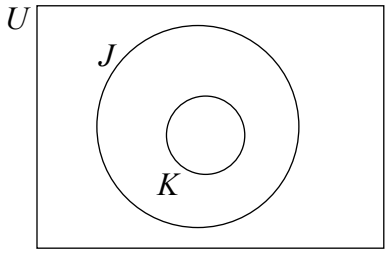
(iii) Diagram 3. [4]

(b) Shade, on the Venn diagram, the region represented by the set

(i) $(H \cup I)'$;



(ii) $J \cap K$. [2]



(This question continues on the following page)

(Question 9 continued)

Working:

Answers:

(a) (i) . . . . . .

(ii) . . . . . .

(iii) . . . . . .

10. The following function models the growth of a bacteria population in an experiment,

$$P(t) = A \times 2^t,\ t \geq 0$$

where A is a constant and t is the time, in hours, since the experiment began.

Four hours after the experiment began, the bacteria population is 6400.

(a) Find the value of A. [2]

(b) Interpret what A represents in this context. [1]

(c) Find the time since the experiment began for the bacteria population to be equal to $40A$. [3]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

. . . . . .

(c) . . . . . .

11. Consider the graph of the function $f(x) = \frac{3}{x} - 2$, $x \neq 0$.

(a) Write down the equation of the vertical asymptote. [2]

(b) Write down the equation of the horizontal asymptote. [2]

(c) Calculate the value of x for which $f(x) = 0$. [2]

Working:

Answers:

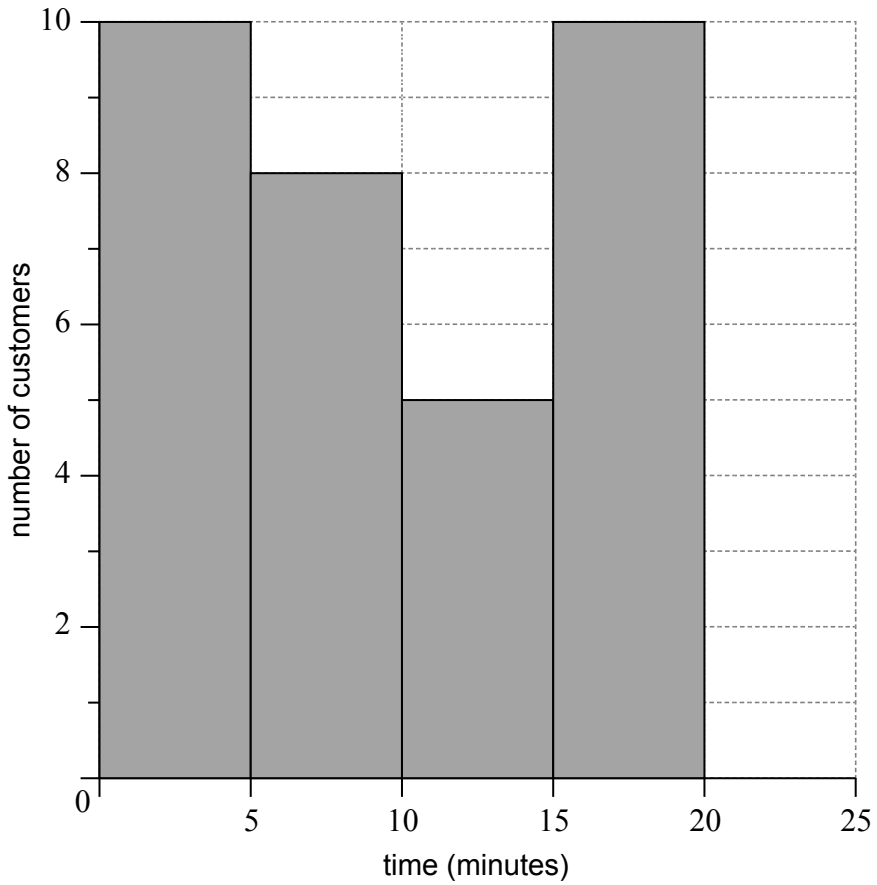
(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

12. The histogram shows the time, t, in minutes, that it takes the customers of a restaurant to eat their lunch on one particular day. Each customer took less than 25 minutes**.**

The histogram is incomplete, and only shows data for $0 \leq t < 20$.



(a) Write down the mid-interval value for $10 \leq t < 15$. [1]

The mean time it took **all** customers to eat their lunch was estimated to be 12 minutes.

It was found that k customers took between 20 and 25 minutes to eat their lunch.

(b) (i) Write down the total number of customers in terms of k.

(ii) Calculate the value of k. [4]

(c) Hence, complete the histogram. [1]

(This question continues on the following page)

(Question 12 continued)

Working:

Answers:

(a) . . . . . .

(b) (i) . . . . . .

(ii) . . . . . .

13. A factory produces shirts. The cost, C, in Fijian dollars (FJD), of producing x shirts can be modelled by

$$C(x) = (x - 75)^2 + 100.$$

(a) Find the cost of producing 70 shirts. [2]

The cost of production should not exceed 500 FJD. To do this the factory needs to produce at least 55 shirts and at most s shirts.

(b) Find the value of s. [2]

(c) Find the number of shirts produced when the cost of production is lowest. [2]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

14. Consider the function $f(x) = \frac{x^4}{4}$.

(a) Find $f'(x)$. [1]

(b) Find the gradient of the graph of f at $x = -\frac{1}{2}$. [2]

(c) Find the x-coordinate of the point at which the **normal** to the graph of f has gradient $-\frac{1}{8}$. [3]

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

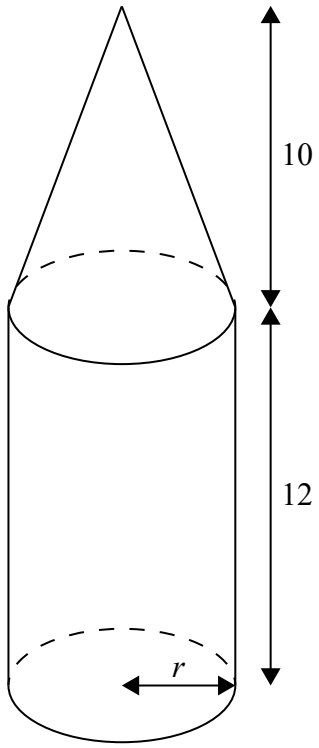
(c) . . . . . .

15. Julio is making a wooden pencil case in the shape of a large pencil. The pencil case consists of a cylinder attached to a cone, as shown.

The cylinder has a radius of r cm and a height of 12 cm.

The cone has a base radius of r cm and a height of 10 cm.

diagram not to scale



(a) Find an expression for the slant height of the cone **in terms of** r. [2]

The total external surface area of the pencil case rounded to 3 significant figures is $570\,\text{cm}^2$.

(b) Using your graphic display calculator, calculate the value of r. [4]

(This question continues on the following page)

(Question 15 continued)

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.